

Part 2: Question 1 (20 points)

Consider an economy with $I = 2$ consumers and $L = 2$ goods. The utility functions for both consumers are:

$$u_1(\mathbf{x}_1) = -\frac{4}{x_{11}} + x_{21}$$
$$u_2(\mathbf{x}_2) = -\frac{16}{x_{12}} + x_{22}$$

where $x_{\ell i}$ represents consumer i 's consumption of the ℓ th good. The price of good 2 is normalized to 1 and the price of good 1 is p . Each consumer has no endowment of good 1 and an initial endowment of good 2 equal to ω_{2i} where $\omega_{2i} > \sqrt{8}$ for $i = 1, 2$. Each consumer's consumption set is $X_i = \mathbb{R}_+^2$.

There are 3 competitive firms that can produce good 1 from good 2 according to the following production technology:

$$\mathcal{Y}_j = \left\{ \mathbf{y}_j \in \mathbb{R}^2 : y_{1j} \leq (-3y_{2j})^{\frac{1}{3}}, y_{2j} \leq 0 \right\}$$

In words: z units of good 2 can produce up to $(3z)^{\frac{1}{3}}$ units of good 1. Each consumer has an equal share in the profits of each firm.

- (i) [4 points] Find the aggregate supply function.
- (ii) [4 points] Find the profit function of each firm.
- (iii) [4 points] Find the aggregate excess demand function.
- (iv) [4 points] Show that the aggregate excess demand function **does not** have the gross-substitute property.
- (v) [4 points] Find the equilibrium price of good 1.

Part 2: Question 2 (20 points)

Consider a pure exchange economy with L goods and I consumers with preferences \succsim_i over $X_i = \mathbb{R}_+^L$ such that the aggregate excess demand function $\mathbf{z}(\mathbf{p})$ is continuous over all $\mathbf{p} \in \mathbb{R}_+^L$ and satisfies:

- $\mathbf{z}(\mathbf{p}) = \mathbf{z}(\alpha\mathbf{p})$ for all $\alpha > 0$ (homogeneity of degree zero).
- $\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) \leq 0$ for all $\mathbf{p} \in \mathbb{R}_+^L$.

Notice that Walras' law is not satisfied for all price vectors. We do not have local nonsatiation.

The distribution of the endowment vector satisfies $\omega_i \gg \mathbf{0}$ for all i . There is a single firm whose production set is:

$$Y_1 = \{\mathbf{y}_1 \in \mathbb{R}^L : \mathbf{y}_1 \leq \mathbf{0}\}$$

Each consumer i is entitled to a share $\theta_{i1} = \frac{1}{I}$ of the firm's profits.

Define the following function $\mathbf{f} : \Delta \rightarrow \Delta$:

$$\mathbf{f}(\mathbf{p}) = \{f_\ell(\mathbf{p})\}_{\ell=1}^L = \left\{ \frac{p_\ell + \max\{0, z_\ell(\mathbf{p})\}}{1 + \sum_{k=1}^L \max\{0, z_k(\mathbf{p})\}} \right\}_{\ell=1}^L$$

where:

$$\Delta = \left\{ \mathbf{p} \in \mathbb{R}_+^L : \sum_{\ell=1}^L p_\ell = 1 \right\}$$

- [2 points]** Show that Δ is non-empty.
- [2 points]** Show that Δ is convex.
- [2 points]** Show that Δ is compact.
- [2 points]** Show that if $\mathbf{p} \in \Delta$, then $\mathbf{f}(\mathbf{p}) \in \Delta$.
- [2 points]** Provide some argumentation that $\mathbf{f} : \Delta \rightarrow \Delta$ is continuous. You do not need to give a full rigorous proof.
- [5 points]** Prove that there exists a price vector $\mathbf{p}^* \geq \mathbf{0}$, $\mathbf{p}^* \neq \mathbf{0}$ that satisfies $\mathbf{z}(\mathbf{p}^*) \leq \mathbf{0}$.
- [5 points]** If there is a price vector $\mathbf{p}^* \geq \mathbf{0}$, $\mathbf{p}^* \neq \mathbf{0}$ that satisfies $\mathbf{z}(\mathbf{p}^*) \leq \mathbf{0}$, is it necessarily a Walrasian equilibrium price vector?